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THE EFFECT OF DIGITIZING NOISE
ON SPECTRAL SIGNAL TO NOISE RATIOS
IN FOURIER SPECTROSCOPY

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ABSTRACT

A discussion of digitizing noise and its effect on spectral signal to noise ratios subject to the limitations of the telemetry system forms the main body of this study. Simulated and actual test data are compared with the theoretical calculations and an optimum working code is derived. The code used in the Nimbus B Infrared Interferometer Spectrometer is found to be very close to the optimum condition for the number of bits used per data word.

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THE EFFECT OF DIGITIZING NOISE ON SPECTRAL SIGNAL TO NOISE RATIOS IN FOURIER SPECTROSCOPY

I. INTRODUCTION

The purpose of this study is to determine the number of bits necessary to represent each data point of the record sent by the Infrared Interferometer Spectrometer (IRIS) instrumentation such that the signal to noise ratio in the spectrum due to digitizing effects is kept to a minimum, and that the number of bits per word is such that the total number transmitted stays within existing telemetry limits. The study will be conducted in two parts; a theoretical description of the effect, and computer simulation for a test spectrum which consists of emission and absorption lines. A study of thermal or random noise effects is also included in combination with the digitizing noise.

The primary instrument in the IRIS experiment is a Michelson interferometer which produces a record from which it is possible to obtain a spectrum of a source by calculating the Fourier transform of the record which is recorded as a function of time or distance. This method of obtaining a spectrum known as Fourier spectroscopy and techniques for this are discussed extensively in the literature.^{①②} In this study, we are primarily concerned with the types of noise on this record, and how the noise transforms in the spectrum.

The two types of noise that are most common on the recording (henceforth called the interferogram) are (1) detector noise due to electron movement within the detector and which is a function of temperature, but not a function of incident radiation, and (2) digitizing or quantum noise which is caused by sampling the amplitudes at discrete levels. The latter is a function of the analog to digital system used, and on ground is not a serious problem since five or six place decimal A to D converters are available, however, on satellites a restriction on the accuracy is imposed by the telemetry system. For instance, with the present Nimbus B system, the IRIS telemetry has a transmission rate of 3.75 kilobits per second. For an 11 second scan and 3408 words, this yields 12 binary bits per word of which 4 bits used are for housekeeping. Hence, one is limited to sampling 256 discrete levels of voltage. The round off error due to this arrangement may be considerable even if one has a noiseless detector. This error essentially introduces noise on the interferogram of a type similar to that of the detector which is assumed to be white noise.

II. NOISE THEORY

We now consider the question of the computation of the signal to noise ratio in the spectrum for a given signal to noise ratio in the interferogram. The expression to be derived makes the assumption that the noise in the interferogram is uncorrelated to the signal.

The autocorrelation theorem ^③ for aperiodic functions is

$$\phi(\tau) = \int_{-\infty}^{\infty} f(x) f(x + \tau) dx = \int_{-\infty}^{\infty} |F(\sigma)|^2 d\sigma \quad (1)$$

where $F(\sigma)$ is the Fourier transform of $f(t)$ and is related by the transform pairs

$$F(\sigma) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi\sigma x} dx \quad (2a)$$

and

$$f(x) = \int_{-\infty}^{\infty} F(\sigma) e^{i2\pi\sigma x} d\sigma \quad (2b)$$

In the above notation $f(x)$ corresponds to the interferogram or noise record or a linear combination of the two, and $F(\sigma)$ is the complex spectrum assuming $f(x)$ is real. From Equation (2b) the following property may be deduced:

$$f(0) = \int_{-\infty}^{\infty} F(\sigma) d\sigma \quad (3)$$

which means that the amplitude of the interferogram at zero path-difference is equal to the total spectral intensity. Also, from Equation (1) at $\tau = 0$ we obtain

$$\int_{-\infty}^{\infty} f^2(x) dx = \int_{-\infty}^{\infty} |F(\sigma)|^2 d\sigma \quad (4a)$$

which states that the square of the interferogram equals the square of the spectrum, that is, the energy densities of the two are equal. Assuming finite limits and discrete sampling with distance or time at about the Nyquist rate of two samples per highest frequency (Δ), we can write

$$\int_{-X}^X f^2(x) dx \approx \Delta \sum_{i=-N}^N f^2(i\Delta)$$

$$\int_{-\sigma_m}^{\sigma_m} |F(\sigma)|^2 d\sigma \approx \frac{1}{N\Delta} \sum_{j=-M_n}^{M_n} \left| F\left(\frac{j}{N\Delta}\right) \right|^2$$

where $dx \rightarrow \Delta$, $X = N\Delta$, $\sigma_m = 1/\Delta$, $d\sigma \rightarrow 1/N\Delta$, and Equation (4a) becomes

$$2NX \sum_{i=-N}^N \frac{f^2(i\Delta)}{2N} = 2M_n \sigma_m \sum_{j=-M_n}^{M_n} \frac{\left| F\left(\frac{j}{N\Delta}\right) \right|^2}{2M_n} \quad (4b)$$

The sums over i and j are the mean square values of the interferogram and spectrum respectively. Considering only a noise record and using n_I^2 and n_S^2 for the mean square values, we can write

$$n_S = n_I \sqrt{\frac{NX}{M_n \sigma_m}} \quad (5)$$

where n_I and n_S are the root mean square values of the recorded noise and its transform respectively.

We can define the signal to noise ratio in the interferogram as the peak signal (which occurs at zero retardation) divided by the rms noise in the interferogram. For the spectral domain we can consider an average signal \bar{S} which is the total spectral intensity divided by the spectral bandwidth. Referring to Equation (3)

$$\bar{S} = \frac{\int_{-\infty}^{\infty} F(\sigma) d\sigma}{2\sigma_m} = \frac{f(0)}{2m_s \frac{1}{N\Delta}} \quad (6)$$

where m_s is the number of resolved spectral elements in the spectral bandwidth, hence the signal to noise ratio in the spectrum becomes

$$\left(\frac{S}{N}\right)_s = \frac{\bar{S}}{n_s} = \frac{N\Delta f(0)}{2M_n n_I} \sqrt{\frac{M_n \sigma_m}{NX}} = \left(\frac{S}{N}\right)_I \sqrt{\frac{1}{4M_n}} \quad (7)$$

Equation (7) assumes that the noise and spectral bandwidths are equivalent. If the noise bandwidth is greater than the spectral bandwidth, Equation (7) becomes

$$\left(\frac{S}{N}\right)_s = \left(\frac{S}{N}\right)_I \sqrt{\frac{M_n}{4m_s^2}} \quad (8)$$

where M_n is the number of resolvable elements in the noise bandwidth. The above assumes unity gain in the electronics hence, if we have a gain G , the peak signal becomes

$$f_G(0) = f(0)G = \frac{2m_s \bar{S}G}{N\Delta} \quad (9)$$

The mean square noise on the interferogram may be considered to be the sum of the mean square values of the digitizing and detector noise respectively,

since they are uncorrelated. As the gain affects only the detector noise we obtain

$$n_I = \sqrt{n_1^2 + G^2 n_2^2} \quad (10)$$

where n_1 and n_2 are the digitizing and detector noise respectively. With this assumption Equation (5) becomes

$$n_S = \sqrt{n_1^2 + G^2 n_2^2} \sqrt{\frac{NX}{M_n \sigma_m}} \quad (11)$$

If one observes a source with very few lines, G will be large, hence $n_S \approx Gn_2 \sqrt{NX/M_n \sigma_m}$. Using Equation (6b), the average spectral signal will be

$$\bar{SG} = \frac{f_G(0) N \Delta}{2M_n}$$

and

$$\left(\frac{S}{N}\right)_S = \frac{f_G(0)}{Gn_2} \sqrt{\frac{1}{4M_n}} = \left(\frac{S}{N}\right)_I \sqrt{\frac{1}{4M_n}} \quad (12)$$

If n_1 is larger than Gn_2 as may be the case for a many lined or broad spectral source, $n_S = n_1 \sqrt{\sigma_M N/M_2 X}$ (the case for no gain) and

$$\frac{\bar{S}}{n_S} = \left(\frac{S}{N}\right)_I \sqrt{\frac{1}{4M_n}} \quad (13)$$

For the case where $n_1 \approx Gn_2$ we have

$$\frac{\bar{S}}{n_S} \approx \sqrt{\frac{1}{8M_n}} \quad (14)$$

In practice we attempt to keep n_1 less than Gn_2 . As can be seen from Equation (10), a factor of two yields about a 10% error in the estimated spectral signal to noise ratio.

III. A MODEL FOR DIGITIZING NOISE

In practice, the number of binary bits per data word limits the number of quantizing levels. Let $K = 2^k$ be the number of levels and $2f(0)$ be the peak voltage. This latter is due to the fact that $f(x)$ may be positive or negative with a maximum value of $|f(0)|$. Let $V(jE_0)$ be the assigned value for $f(x)$ if $jE_0 < f(x) \leq (j+1)E_0$ $j = 0, 1, \dots, K-1$, where $2E_0 = 2f(0)/K$. This is shown in Figure 1a.

For the sampling interval Δ , $f(x)$ may in general maybe approximated by a straight line of slope m ^④ as shown in Figure 1b. Thus the error due to quantizing may be represented by

$$\epsilon(x) = mx \quad \text{where} \quad -\frac{E_0}{m} < x \leq \frac{E_0}{m} . \quad (15)$$

The mean square error is then

$$\frac{m}{2E_0} \int_{-E_0/m}^{E_0/m} \epsilon^2(x) dx = \frac{E_0^2}{3} \quad (16)$$

for each data point. The rms error voltage is then $f(0)/K\sqrt{3}$, hence, the signal to noise ratio in the interferogram for digitizing noise alone is

$$\left(\frac{S}{N}\right)_D = 1.732 \times 2^k . \quad (17)$$

Another scheme for quantizing the interferogram is to quantize points less than a certain percentage of full scale to 2^k and use a coarser quantization for

the remaining points. For instance, Nimbus B quantizes the points of the interferogram that are less than 10% of the maximum to 256 levels. The remaining points are then divided by 10 and quantized, the operation being recorded by one bit of the telemetry data word. The mean square noise due to this is

$$y \frac{(E_0')^2}{3} + (1 - y) \frac{E_0^2}{3}$$

where y is the percentage of total points in the interferogram that are digitized with $2E_0'$ as the step size. The coarse quantization step size is $2E_0$. Since

$$E_0 = \frac{f(0)}{2^k} \quad \text{and} \quad E_0' = \frac{pf(0)}{2^k} \quad p \leq 1 \quad (18)$$

the rms noise becomes

$$\frac{E_0}{\sqrt{3}} \sqrt{y(p^2 - 1) + 1}$$

and the signal to noise ratio due to digitizing is

$$\frac{1.732 \cdot 2^k}{\sqrt{y(p^2 - 1) + 1}} = \left(\frac{S}{N} \right)'_D \quad (19)$$

where the denominator is referred to as the increase factor. Since y and p are always less than one, an increase in S/N ratio results with the same number of bits. As an example if 90% of the words are less than 10% of the maximum value we have an increase of a factor of 3 over equation (12). Figure 2 shows a plot of the increase factor as a function of p .

IV. TEST RESULTS

A model spectrum ^⑤ consisting of emission and absorption lines was used to generate a symmetric interferogram which was then quantized to different levels and retransformed. Figure 3a shows the correct spectrum; Figures 3b-3d show the spectrum of interferograms normalized to 256, 1024, and 2048 levels ($k = 8, 10, 11$); Figure 3e shows the spectrum computed by applying the Nimbus B scheme. The noise in this spectrum lies somewhere between that of Figure 3c and 3d. A look at the interferogram revealed that $y \approx .98$. From Figure 2 the (S/N) increase is about 5 and corresponds to 2500 sampled levels.

Figure 4 shows the total percentage of points falling below a given percentage of the maximum voltage accepted by the IRIS system. The data is the average of 13 interferograms looking at the atmosphere obtained by the University of Michigan balloon experiment. An interesting feature of this is that the Nimbus B acquisition system is nearly optimal as about 98% of the data points are less than 10% of the maximum.

V. SUMMARY

The formula for determining the signal to noise ratio in the spectral domain is found as a function of the signal to noise ratio of the interferogram and is given by equation 8. A general technique for comparing various digitizing schemes is given, and it is found that the Nimbus B IRIS scheme is near the optimum. In general, with proper digitizing codes, quantization noise is not a serious consideration in the signal to noise ratio of the spectrum.

VI. ACKNOWLEDGMENT

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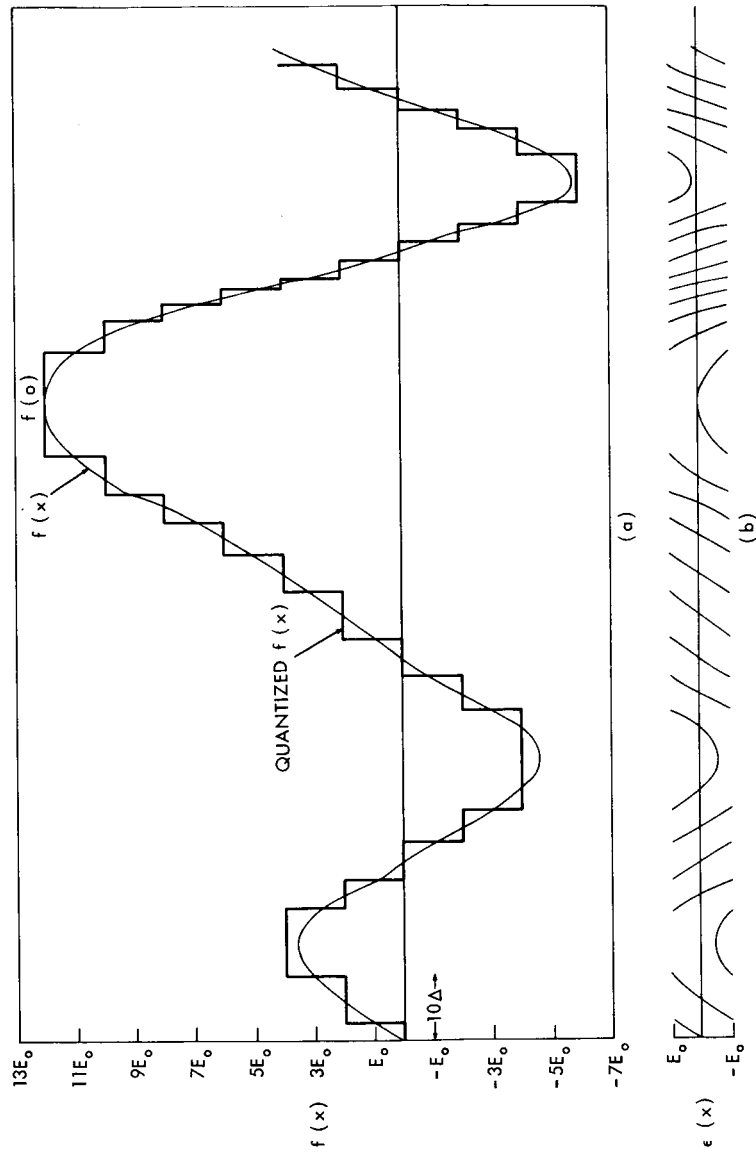


Figure 1-(a) Staircase quantizer; (b) Error due to staircase quantization.

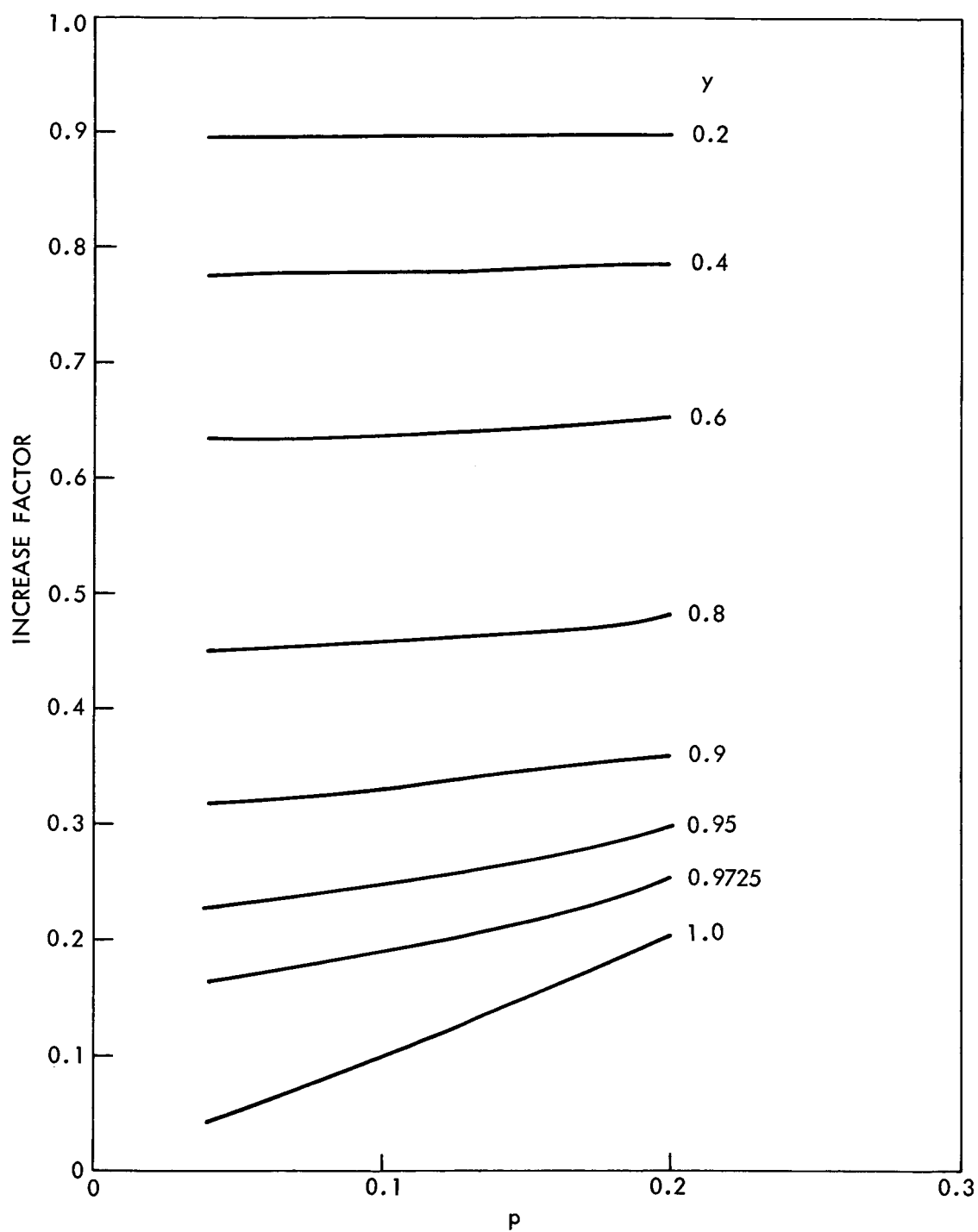
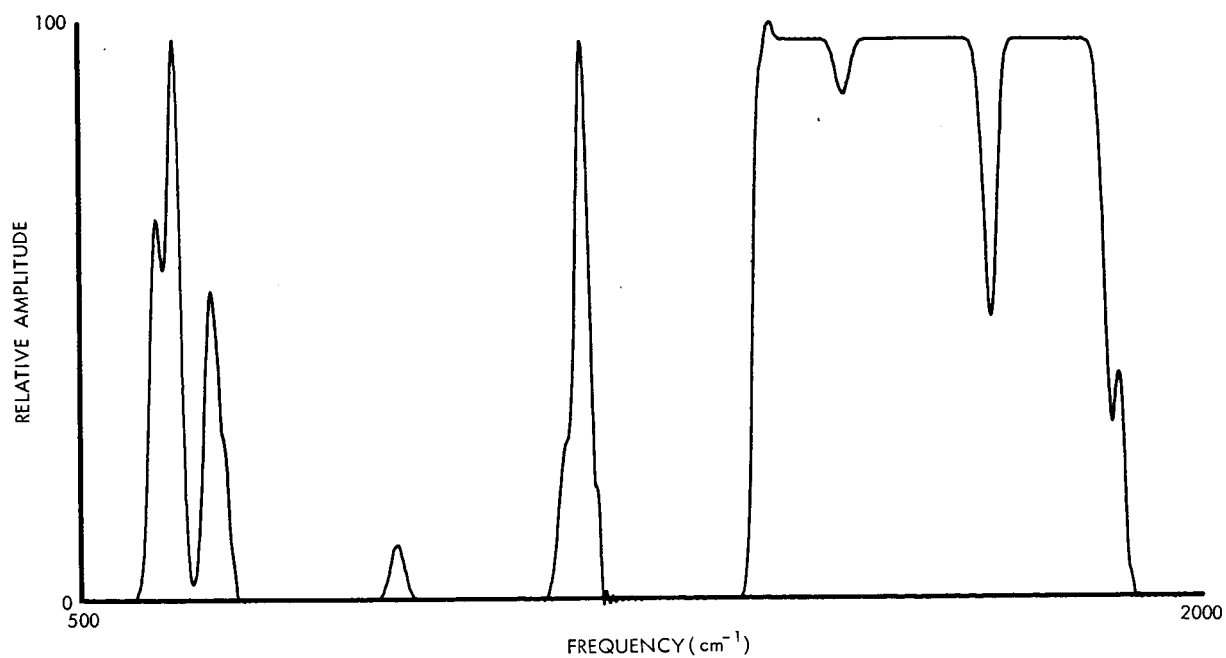
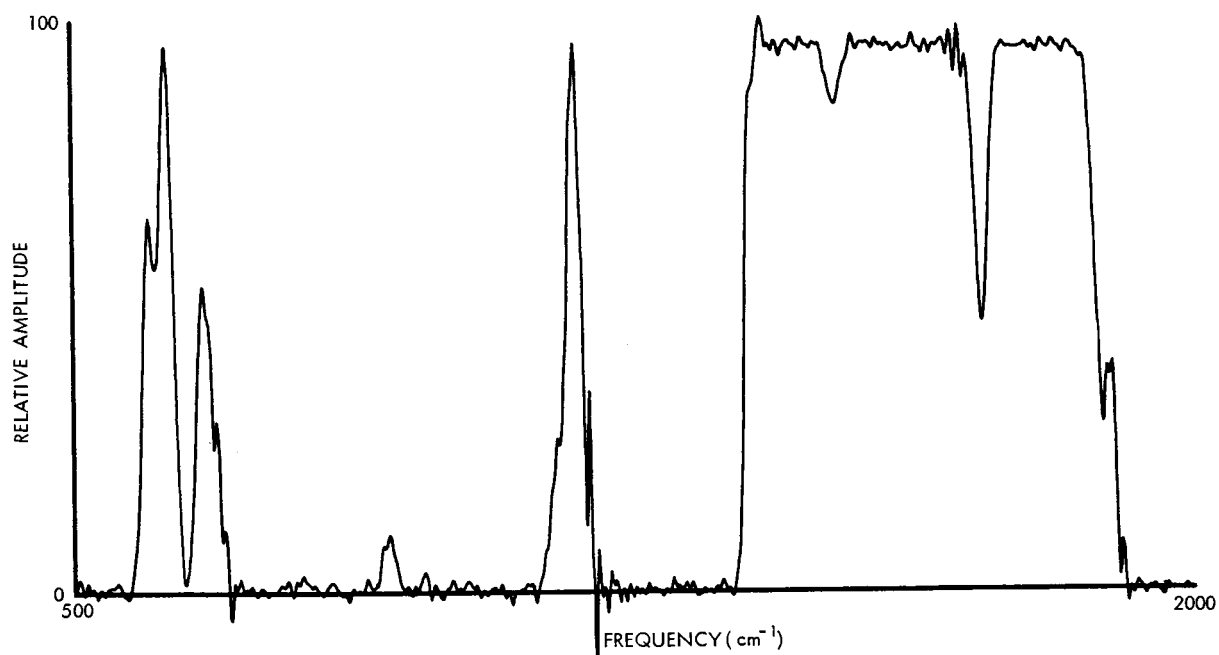


Figure 2-Signal to Noise increase factor $\left(\sqrt{\gamma(p^2 - 1)} + 1 \right)$ vs p (fraction of $f(0)$).

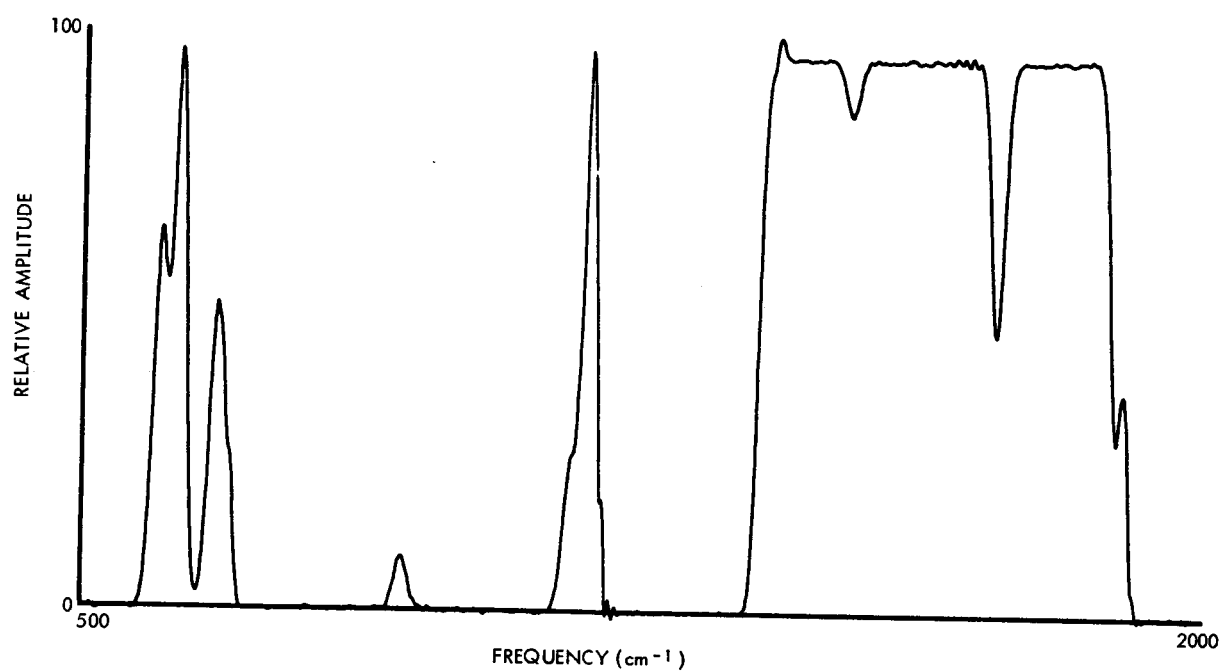


(a) No quantizing

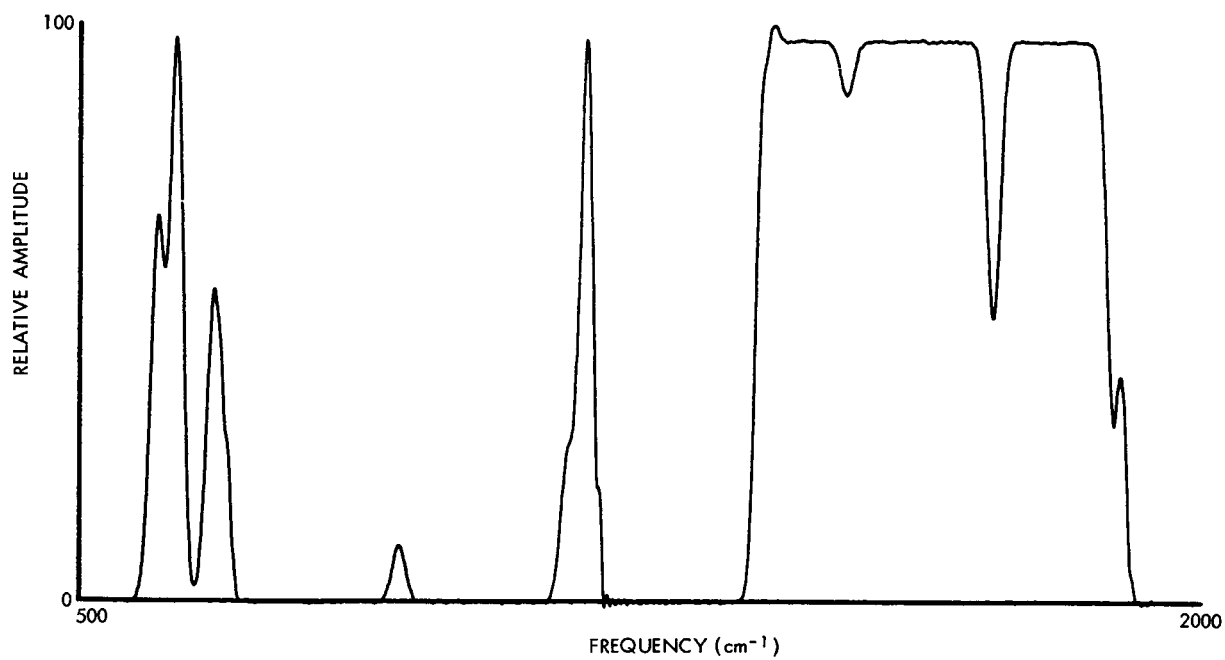


(b) Quantized to 256

Figure 3-Test spectrum with interferogram quantizing effects.

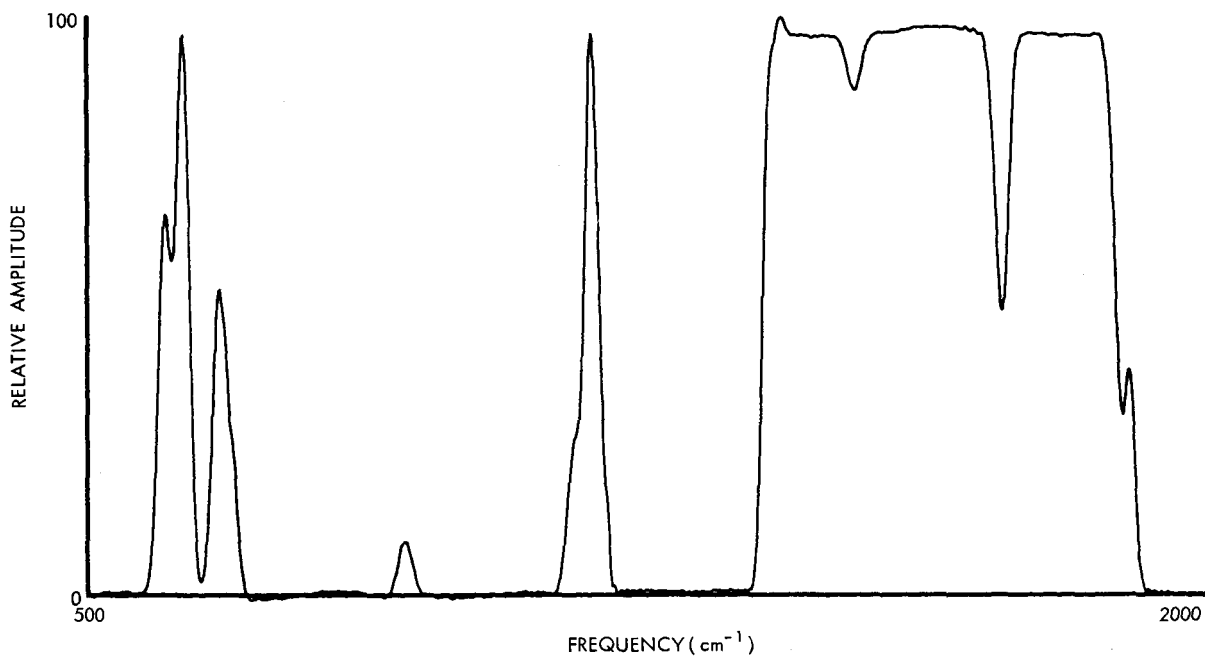


(c) Quantized to 1024



(d) Quantized to 2048

Figure 3-Test spectrum with inteferogram quantizing effects.



(e) Nimbus B quantizing

Figure 3-Test spectrum with inteferogram quantizing effects.

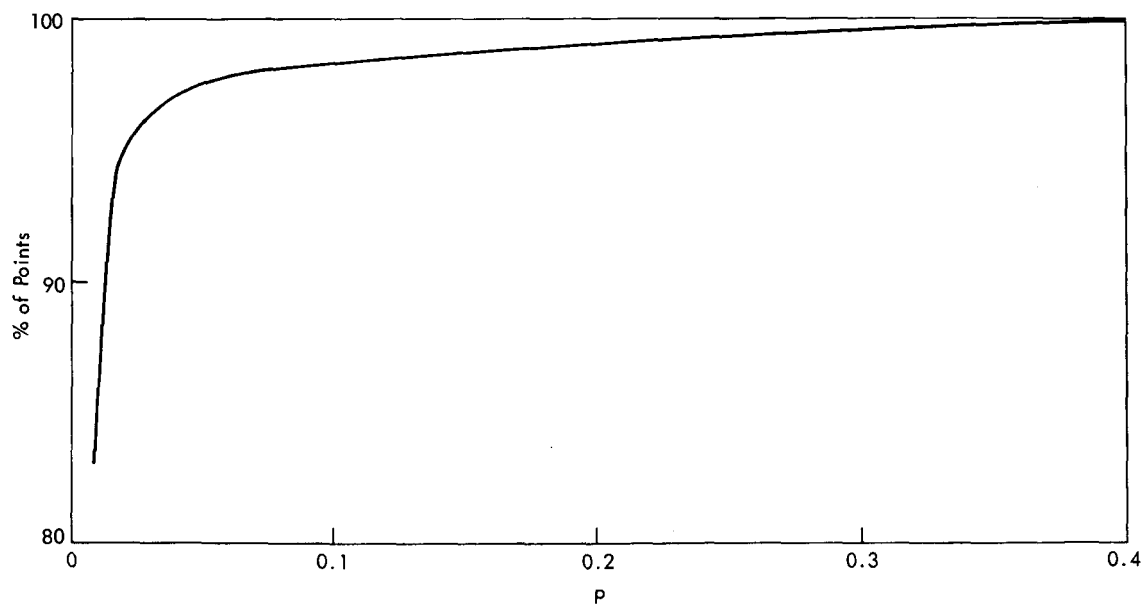


Figure 4-Percentage of data points falling below a given fraction of maximum allowable signal vs p.